# Sheet 2 (will not be marked)

**1.** If $z = e^{xy}$ and $x = 2u + v$, $y = \frac{u}{v}$ find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ in terms of $u$ and $v$ using the chain rule.

**2.** Let $z = f(x - y, y - x)$. Show that $\partial z/\partial x + \partial z/\partial y = 0$.

**3.** Find the stationary points of the function

$$f(x, y) = y^2 + xy + 3y + 2x + 3$$

and determine their nature.

**4.** Find the stationary points of the function

$$f(x, y) = xy^2 - 2y^3 - x^2 + 2xy$$

and determine their nature.

**5.** Find the stationary points of the function

$$f(x, y) = x^2 + y^2 + \frac{2}{xy}$$

and determine their nature.

**6.** Find the dimensions of the most economical open-top rectangular crate $96\,\mathrm{m}^3$ in volume when the base costs 30 pence per square meter and the sides cost 10 pence per square meter. What is the actual minimum cost ?

**Answers/Hints:**

**1.** $\frac{\partial z}{\partial u} = \left(\frac{4u}{v} + 1\right) e^{(2u+v)(u/v)}$, $\frac{\partial z}{\partial v} = -\frac{2u^2}{v^2} e^{(2u+v)(u/v)}$.

**2.** Introduce intermediate variables $u = x - y$ and $v = y - x$.

**3.** $(1, -2)$ saddle.

**4.** $(0, 0)$ saddle, $(\frac{3}{2}, 1)$ maximum, $(4, 2)$ saddle.

**5.** $(1, 1)$ and $(-1, -1)$, both minimum.

**6.** $4 \times 4 \times 6$. Cost is £14.40.