Mathematics 1b: Sheet 5 (will not be marked)

1. Solve the following differential equations for y, expressing the answer in the from $y = f(x)$:

(a) $\dfrac{dy}{dx} = \dfrac{\sinh 4x}{y^4}$

(b) $\dfrac{dy}{dx} - (x+1)y^2 = 0$

(c) $(1+x)\dfrac{dy}{dx} = 4y$ given that $y = 1$ when $x = 1$

(d) $\dfrac{dy}{dx} - e^{2x}(y^2+1) = 0$ subject to $y(0) = 1$

(e) $\dfrac{dy}{dx} = y(y-1)$ subject to $y(0) = 2$

2. If two chemicals are mixed together then the differential equation

$$\frac{dy}{dt} = k(N-y)(M-y)$$

can be used to describe the reaction which results in the formation of another chemical with concentration $y(t)$, where k is a positive constant and M, N are the numbers of molecules of the two chemicals before mixing (assume $M \neq N$). Solve the above differential equation to obtain y as a function of t subject to the condition $y(0) = 0$.
Use your solution to determine $\lim_{t \to \infty} y(t)$, distinguishing between the two cases (i) $M > N$ and (ii) $M < N$.

3. For very hot objects that lose heat by radiation it is necessary to use *Stefan's law of cooling*, rather than Newton's law of cooling. According to Stefan's law, the rate of change of temperature of a body due to radiation of heat is given by

$$\frac{dT}{dt} = k(M^4 - T^4)$$

where k is some constant, T is the temperature of the body in degrees Kelvin and M is the (constant) temperature of the surrounding medium in Kelvin.

(i) Factorise $M^4 - T^4$ and express $\dfrac{1}{M^4 - T^4}$ in partial fractions.

(ii) Show that

$$\ln\left(\frac{M+T}{M-T}\right) + 2\tan^{-1}\frac{T}{M} = 4kM^3t + c$$

where c is a constant.