Mathematics 1b: Sheet 9 (will not be marked)

1. Find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 1 & 2 & 5 \end{pmatrix}$$

and use this inverse to solve the system of equations

$$\begin{aligned} x_1 - x_3 &= 2 \\ 2x_1 + x_2 - x_3 &= 4 \\ x_1 + 2x_2 + 5x_3 &= 14 \end{aligned}$$

2. Let

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix}$$

Evaluate A^2 and A^3 and show that

$$A^3 - A^2 - 5A + 8I = 0$$

where I denotes the 3×3 identity matrix and 0 denotes the 3×3 zero matrix. Hence show that $A^{-1} = \frac{1}{8}(5I + A - A^2)$ and use this formula to evaluate A^{-1}.

3. Solve the system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 2x_1 + x_2 &= -1 \\ -2x_1 + 3x_2 + 8x_3 &= 13 \end{aligned}$$

4. For each of the following matrices find its determinant and state whether the matrix is singular or not.

$$\text{(i)} \quad \begin{pmatrix} 2 & -1 & 7 \\ 4 & 6 & -2 \\ 5 & 8 & -3 \end{pmatrix} \qquad \text{(ii)} \quad \begin{pmatrix} -5 & 2 & 4 \\ 1 & -3 & 4 \\ 0 & 1 & 3 \end{pmatrix} \qquad \text{(iii)} \quad \begin{pmatrix} 2 & 5 & -3 \\ 3 & -2 & 1 \\ 7 & 8 & -5 \end{pmatrix}$$

5. Show that

$$\begin{vmatrix} 2x + y + z & x & x^2 \\ x + 2y + z & y & y^2 \\ x + y + 2z & z & z^2 \end{vmatrix} = (x + y + z)(x - y)(y - z)(z - x)$$

(NB: Don't just expand it out! Use the various properties of determinants.)

Answers on back

Answers:

1. $A^{-1} = \frac{1}{4}\begin{pmatrix} 7 & -2 & 1 \\ -11 & 6 & -1 \\ 3 & -2 & 1 \end{pmatrix}$. $\quad x_1 = 5, \quad x_2 = -3, \quad x_3 = 3.$

2. $A^2 = \begin{pmatrix} -2 & -1 & -6 \\ 9 & 5 & 6 \\ 3 & 0 & 8 \end{pmatrix}, \quad A^3 = \begin{pmatrix} -5 & -6 & 4 \\ 24 & 7 & 26 \\ 3 & 5 & -10 \end{pmatrix}, \quad A^{-1} = \frac{1}{8}\begin{pmatrix} 8 & 0 & 8 \\ -6 & 2 & -2 \\ -3 & 1 & -5 \end{pmatrix}.$

3. $x_1 = -2 + \alpha,\ x_2 = 3 - 2\alpha,\ x_3 = \alpha$ for any number α.

4. (i) 8 (ii) 63 (iii) 0 third matrix is singular.