UNIVERSITY OF SURREY©

School of Electronics and Physical Sciences

Undergraduate Programmes in Electronic Engineering

Level 1

Module EE1.MAB Mathematics B

Time allowed: 2 hours — Spring Semester 2006

Answer all questions. All working must be shown.

The marks for each question are shown in brackets. You should note that some questions carry more marks than others.

You may use the **Tables of Constants, Formulae and Transforms**. Approved calculators may be used.

1.

(i) Find the binomial expansion of

$$\frac{1}{(1+x^2)^{\frac{1}{4}}}$$

up to and including the term in x^6. [6]

(ii) Use the expansions of e^x and $\cos x$ (which you may quote from the booklet) to find the Maclaurin expansion of $e^{2x}\cos 2x$ up to and including the term in x^3. [6]

2. Using L'Hopital's rule or otherwise, find the limits

$$\text{(a)} \quad \lim_{x\to 1} \frac{\ln x}{x^4-1} \qquad\qquad \text{(b)} \quad \lim_{x\to 0} \frac{\tan x}{\tan 3x}$$ [5, 5]

3.

(i) Given that $\sinh x = \frac{12}{5}$, find (a) $\cosh x$, (b) $\tanh x$, (c) $\sinh 2x$. [7]

(ii) Using an appropriate substitution, find

$$\int \frac{dx}{\sqrt{x^2-2x+10}}$$

in terms of an inverse hyperbolic function. [6]

4. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = -2y^2x$ [4]

(ii) $\dfrac{dy}{dx} = (y^2+1)x^2$ subject to $y(0) = 1$ [6]

(iii) $\dfrac{dy}{dx} + 3y = x$ [6]

5. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 7\dfrac{dy}{dx} + 10y = 0$ [4]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = x^2 - 1$ [10]

6. Show that the Newton Raphson method, applied to the equation $x^4 + x - 3 = 0$, leads to the recursion formula

$$x_{n+1} = x_n - \frac{x_n^4 + x_n - 3}{4x_n^3 + 1}$$

With an initial guess of $x_0 = 1$, find an approximation to the positive root of the equation, accurate to three decimal places. [8]

7. Evaluate the following double integrals:

(i) $\int_0^{\pi}\int_0^{x} x\sin y\,dy\,dx$ [6]

(ii) $\iint_D (1+xy)\,dA$ where D is the triangle with vertices $(1,1)$, $(2,1)$ and $(2,2)$. [8]

8. Let the 2π periodic function $f(t)$ be defined by

$$f(t) = \begin{cases} 0 & -\pi < t < -\pi/2 \\ \pi & -\pi/2 < t < \pi/2 \\ 0 & \pi/2 < t < \pi \end{cases}$$

with $f(t+2\pi) = f(t)$ for all t.

(i) Draw the graph of $f(t)$ for $-\pi < t < \pi$. [3]

(ii) How do we know that the Fourier series of $f(t)$ will contain only cosine terms? [2]

(iii) Find the Fourier series of $f(t)$ in the form

$$f(t) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nt$$

giving the value of a_0 and a_n for $n = 1, 2, 3, \ldots$. [8]

Internal Examiner: S.A. GOURLEY